Module
3
Design for Strength

Lesson
3
Design for dynamic loading

Instructional Objectives

At the end of this lesson, the students should be able to understand

- Mean and variable stresses and endurance limit.
- S-N plots for metals and non-metals and relation between endurance limit and ultimate tensile strength.
- Low cycle and high cycle fatigue with finite and infinite lives.
- Endurance limit modifying factors and methods of finding these factors.

3.3.1 Introduction

Conditions often arise in machines and mechanisms when stresses fluctuate between a upper and a lower limit. For example in **figure-3.3.1.1**, the fiber on the surface of a rotating shaft subjected to a bending load, undergoes both tension and compression for each revolution of the shaft.
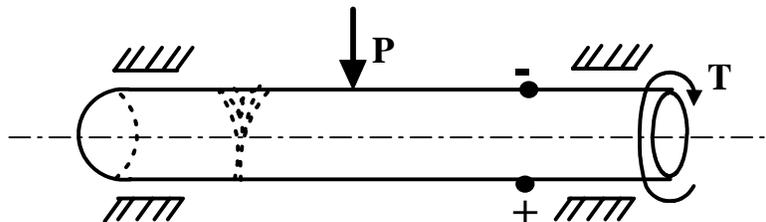
3.3.1.1F- Stresses developed in a rotating shaft subjected to a bending load.

Any fiber on the shaft is therefore subjected to fluctuating stresses. Machine elements subjected to fluctuating stresses usually fail at stress levels much below their ultimate strength and in many cases below the yield point of the material too. These failures occur due to very large number of stress cycle and are known as fatigue failure. These failures usually begin with a small crack

which may develop at the points of discontinuity, an existing subsurface crack or surface faults. Once a crack is developed it propagates with the increase in stress cycle finally leading to failure of the component by fracture. There are mainly two characteristics of this kind of failures:

(a) Progressive development of crack.

(b) Sudden fracture without any warning since yielding is practically absent.

Fatigue failures are influenced by

(i)   Nature and magnitude of the stress cycle.

(ii) Endurance limit.

(iii) Stress concentration.

(iv) Surface characteristics.

These factors are therefore interdependent. For example, by grinding and polishing, case hardening or coating a surface, the endurance limit may be improved. For machined steel endurance limit is approximately half the ultimate tensile stress. The influence of such parameters on fatigue failures will now be discussed in sequence.

3.3.2 Stress cycle

A typical stress cycle is shown in **figure- 3.3.2.1** where the maximum, minimum, mean and variable stresses are indicated. The mean and variable stresses are given by

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$
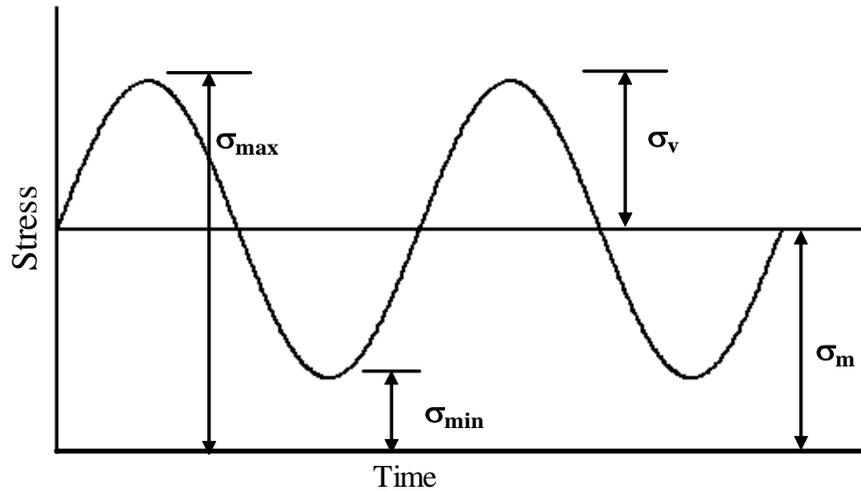
***3.3.2.1F**- A typical stress cycle showing maximum, mean and variable stresses.*

3.3.3 Endurance limit

Figure- 3.3.3.1 shows the rotating beam arrangement along with the specimen.
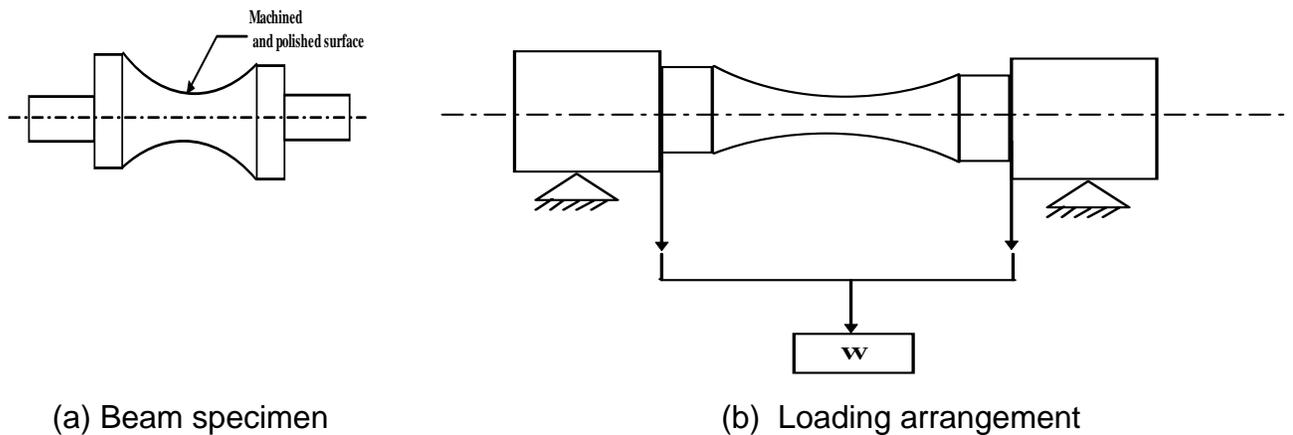
(a) Beam specimen   (b) Loading arrangement

***3.3.3.1F**- A typical rotating beam arrangement.*

The loading is such that there is a constant bending moment over the specimen length and the bending stress is greatest at the center where the section is smallest. The arrangement gives pure bending and avoids transverse shear

since bending moment is constant over the length. Large number of tests with varying bending loads are carried out to find the number of cycles to fail. A typical plot of reversed stress (S) against number of cycles to fail (N) is shown in **figure-3.3.3.2**. The zone below 10^3 cycles is considered as low cycle fatigue, zone between 10^3 and 10^6 cycles is high cycle fatigue with finite life and beyond 10^6 cycles, the zone is considered to be high cycle fatigue with infinite life.
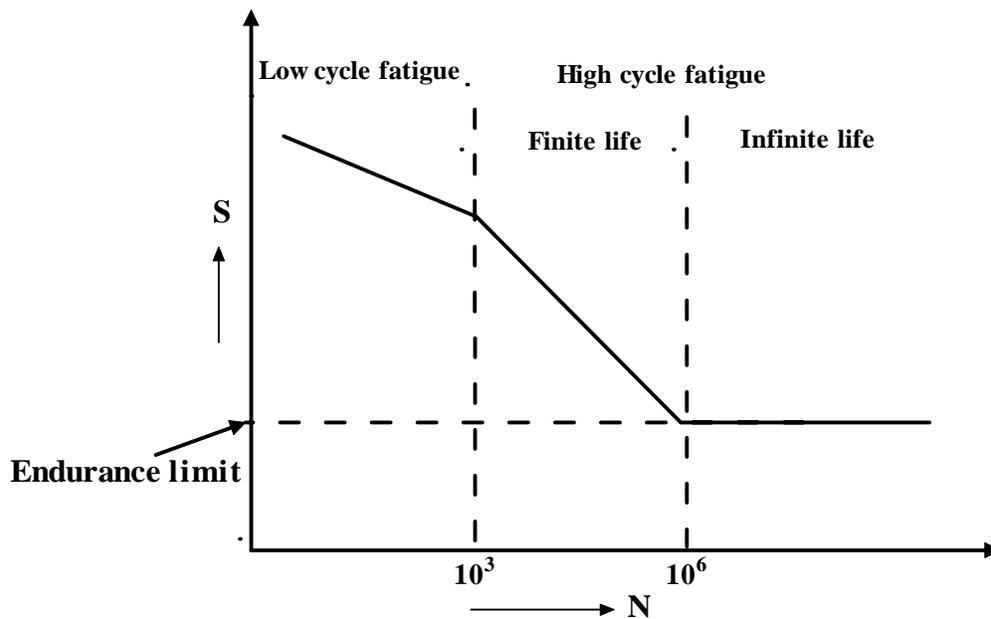
3.3.3.2F*- A schematic plot of reversed stress (S) against number of cycles to fail (N) for steel.*

The above test is for reversed bending. Tests for reversed axial, torsional or combined stresses are also carried out. For aerospace applications and non-metals axial fatigue testing is preferred. For non-ferrous metals there is no knee in the curve as shown in **figure- 3.3.3.3** indicating that there is no specified transition from finite to infinite life.
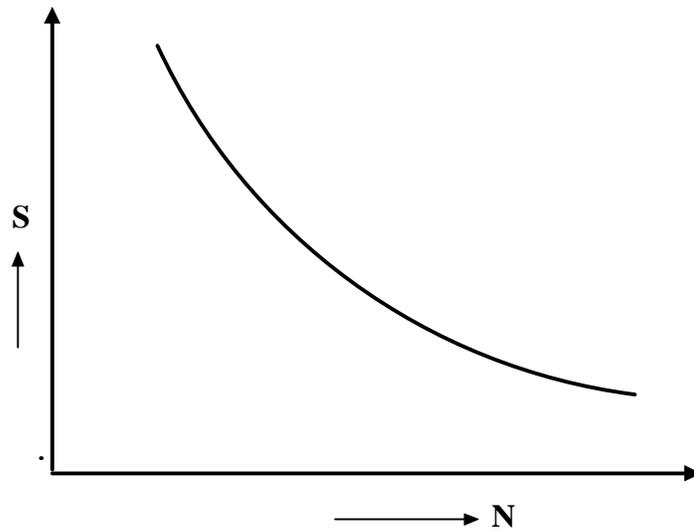
3.3.3.3F- *A schematic plot of reversed stress (S) against number of cycles to fail (N) for non-metals, showing the absence of a knee in the plot.*

A schematic plot of endurance limit for different materials against the ultimate tensile strengths (UTS) is shown in **figure- 3.3.3.4**. The points lie within a narrow band and the following data is useful:

Steel   Endurance limit   ~   35-60 % UTS

Cast Iron   Endurance limit   ~   23-63 % UTS
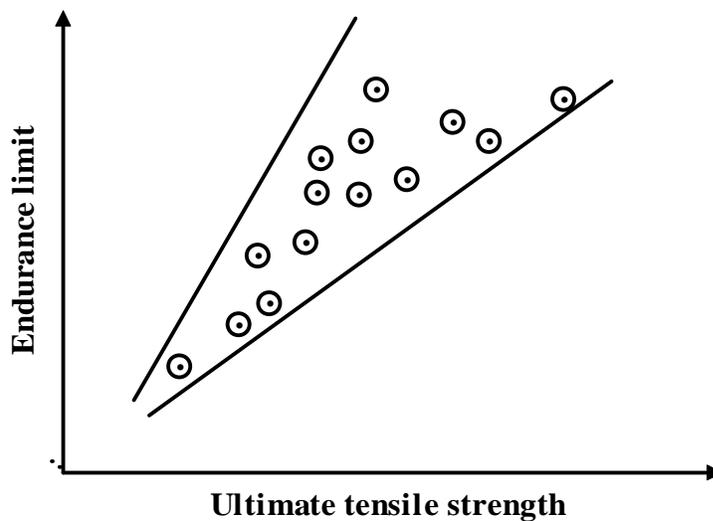
3.3.3.4F- *A schematic representation of the limits of variation of endurance limit with ultimate tensile strength.*

The endurance limits are obtained from standard rotating beam experiments carried out under certain specific conditions. They need be corrected using a number of factors. In general the modified endurance limit σ_e' is given by

$$\sigma_e' = \sigma_e\, C_1 C_2 C_3 C_4 C_5 / K_f$$

C_1 is the size factor and the values may roughly be taken as

$$C_1 = 1, \qquad d \leq 7.6 \text{ mm}$$

$$= 0.85, \qquad 7.6 \leq d \leq 50 \text{ mm}$$

$$= 0.75, \qquad d \geq 50 \text{ mm}$$

For large size $C_1 = 0.6$. Then data applies mainly to cylindrical steel parts. Some authors consider 'd' to represent the section depths for non-circular parts in bending.

C_2 is the loading factor and the values are given as

$$C_2 = 1, \qquad \text{for reversed bending load.}$$

$$= 0.85, \qquad \text{for reversed axial loading for steel parts}$$

$$= 0.78, \qquad \text{for reversed torsional loading for steel parts.}$$

C_3 is the surface factor and since the rotating beam specimen is given a mirror polish the factor is used to suit the condition of a machine part. Since machining process rolling and forging contribute to the surface quality the plots of C_3 versus tensile strength or Brinnel hardness number for different production process, in **figure- 3.3.3.5**, is useful in selecting the value of C_3.
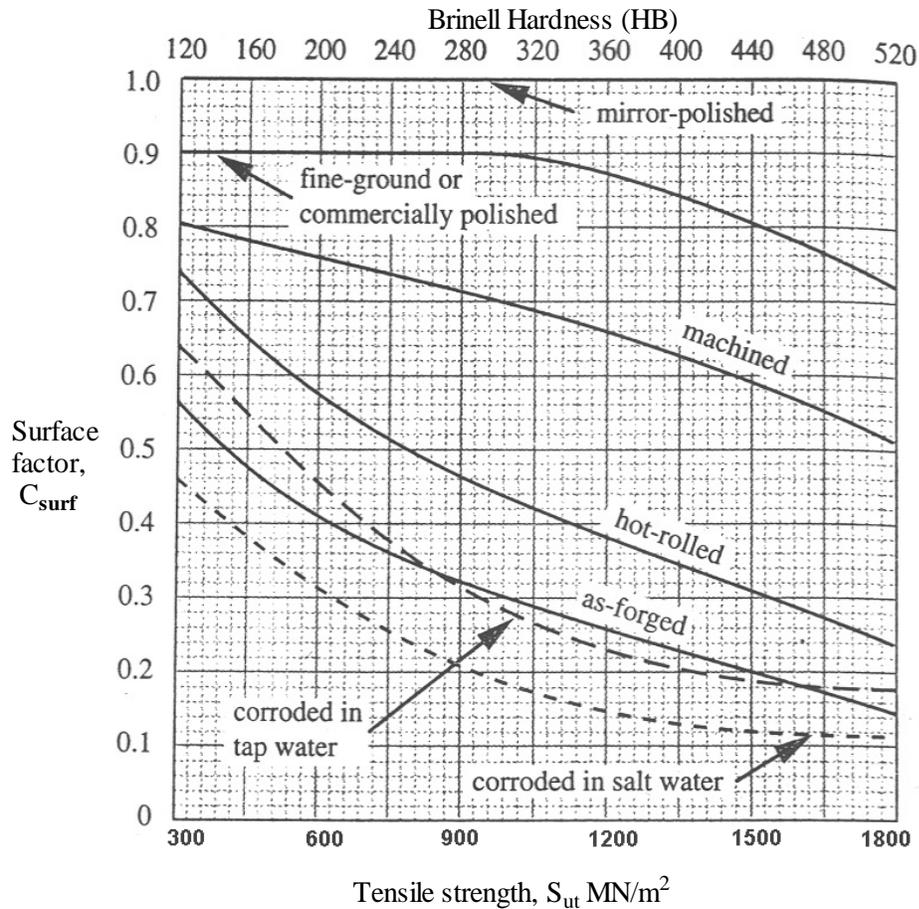
3.3.3.5F- Variation of surface factor with tensile strength and Brinnel hardness for steels with different surface conditions (Ref.[2]).

C_4 is the temperature factor and the values may be taken as follows:

$$C_4 = 1, \qquad \text{for } T \leq 450^{\circ}C.$$

$$= 1\text{-}0.0058(T\text{-}450) \qquad \text{for } 450^{\circ}C < T \leq 550^{\circ}C.$$

C_5 is the reliability factor and this is related to reliability percentage as follows:

Reliability %	C_5
50	1
90	0.897
99.99	0.702

K_f is the fatigue stress concentration factor, discussed in the next section.

3.3.4 Stress concentration

Stress concentration has been discussed in earlier lessons. However, it is important to realize that stress concentration affects the fatigue strength of machine parts severely and therefore it is extremely important that this effect be considered in designing machine parts subjected to fatigue loading. This is done by using fatigue stress concentration factor defined as

$$k_f = \frac{Endurance\ limit\ of\ a\ notch\ free\ specimen}{Endurance\ limit\ of\ a\ notched\ specimen}$$

The notch sensitivity 'q' for fatigue loading can now be defined in terms of K_f and the theoretical stress concentration factor K_t and this is given by

$$q = \frac{K_f - 1}{K_t - 1}$$

The value of q is different for different materials and this normally lies between 0 to 0.7. The index is small for ductile materials and it increases as the ductility decreases. Notch sensitivities of some common materials are given in **table-3.3.4.1** .

3.3.4.1T- Notch sensitivity of some common engineering materials.

Material	Notch sensitivity index
C-30 steel- annealed	0.18
C-30 steel- heat treated and drawn at 480°C	0.49
C-50 steel- annealed	0.26
C-50 steel- heat treated and drawn at 480°C	0.50
C-85 steel- heat treated and drawn at 480°C	0.57
Stainless steel- annealed	0.16
Cast iron- annealed	0.00-0.05
copper- annealed	0.07
Duraluminium- annealed	0.05-0.13

Notch sensitivity index q can also be defined as

$$q = \frac{1}{1 + \left(\dfrac{a}{r}\right)^{1/2}}$$

where, $\sqrt{a}$ is called the Nubert's constant that depends on materials and their heat treatments. A typical variation of q against notch radius r is shown in **figure-3.3.4.2** .
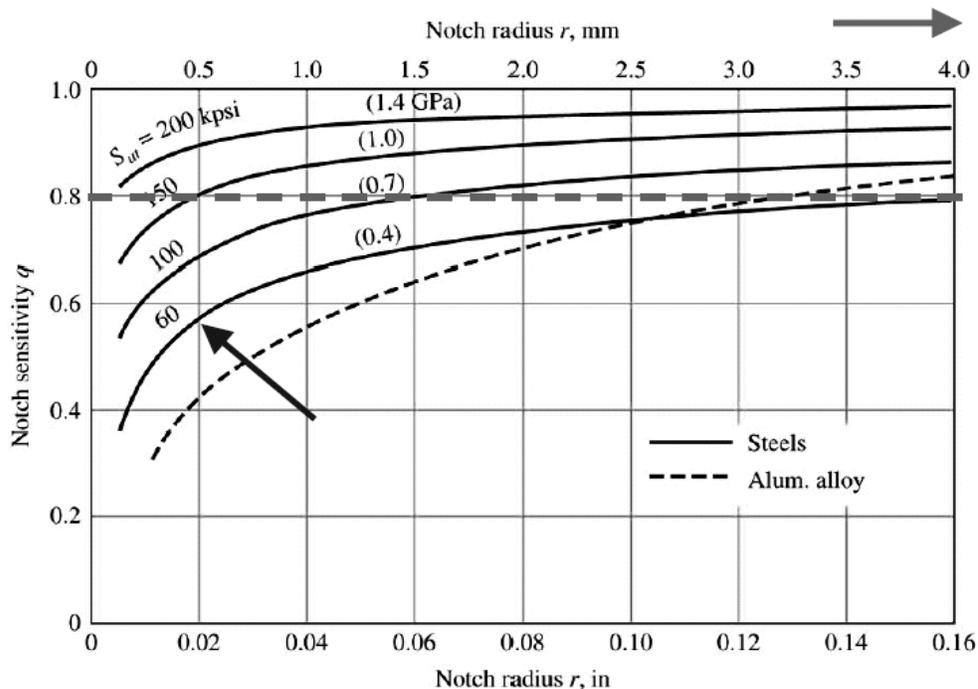
3.3.4.2F- *Variation of notch sensitivity with notch radius for steel and aluminium alloy with different ultimate tensile strengths (Ref.[2]).*

3.3.5   Surface characteristics

Fatigue cracks can start at all forms of surface discontinuity and this may include surface imperfections due to machining marks also. Surface roughness is therefore an important factor and it is found that fatigue strength for a regular surface is relatively low since the surface undulations would act as stress raisers.

It is, however, impractical to produce very smooth surfaces at a higher machining cost.

Another important surface effect is due to the surface layers which may be extremely thin and stressed either in tension or in compression. For example, grinding process often leaves surface layers highly stressed in tension. Since fatigue cracks are due to tensile stress and they propagate under these conditions and the formation of layers stressed in tension must be avoided. There are several methods of introducing pre-stressed surface layer in compression and they include shot blasting, peening, tumbling or cold working by rolling. Carburized and nitrided parts also have a compressive layer which imparts fatigue strength to such components. Many coating techniques have evolved to remedy the surface effects in fatigue strength reductions.

3.3.6 Problems with Answers

Q.1: A rectangular stepped steel bar is shown in **figure-3.3.6.1**. The bar is loaded in bending. Determine the fatigue stress-concentration factor if ultimate stress of the materials is 689 MPa.

r = 5mm
D = 50 mm
d = 40 mm
b = 1 mm
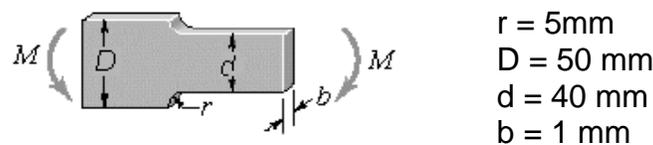
3.3.6.1F

A.1:

From the geometry r/d = 0.125 and D/d = 1.25.

From the stress concentration chart in **figure- 3.2.4.6**

Stress- concentration factor $K_t \approx 1.7$

From **figure- 3.3.4.2**

Notch sensitivity index, $q \approx 0.88$

The fatigue stress concentration factor K_f is now given by

$K_f = 1+q (K_t -1) = 1.616$

3.3.7 Summary of this Lesson

Design of components subjected to dynamic load requires the concept of variable stresses, endurance limit, low cycle fatigue and high cycle fatigue with finite and infinite life. The relation of endurance limit with ultimate tensile strength is an important guide in such design. The endurance limit needs be corrected for a number of factors such as size, load, surface finish, temperature and reliability. The methods for finding these factors have been discussed and demonstrated in an example.